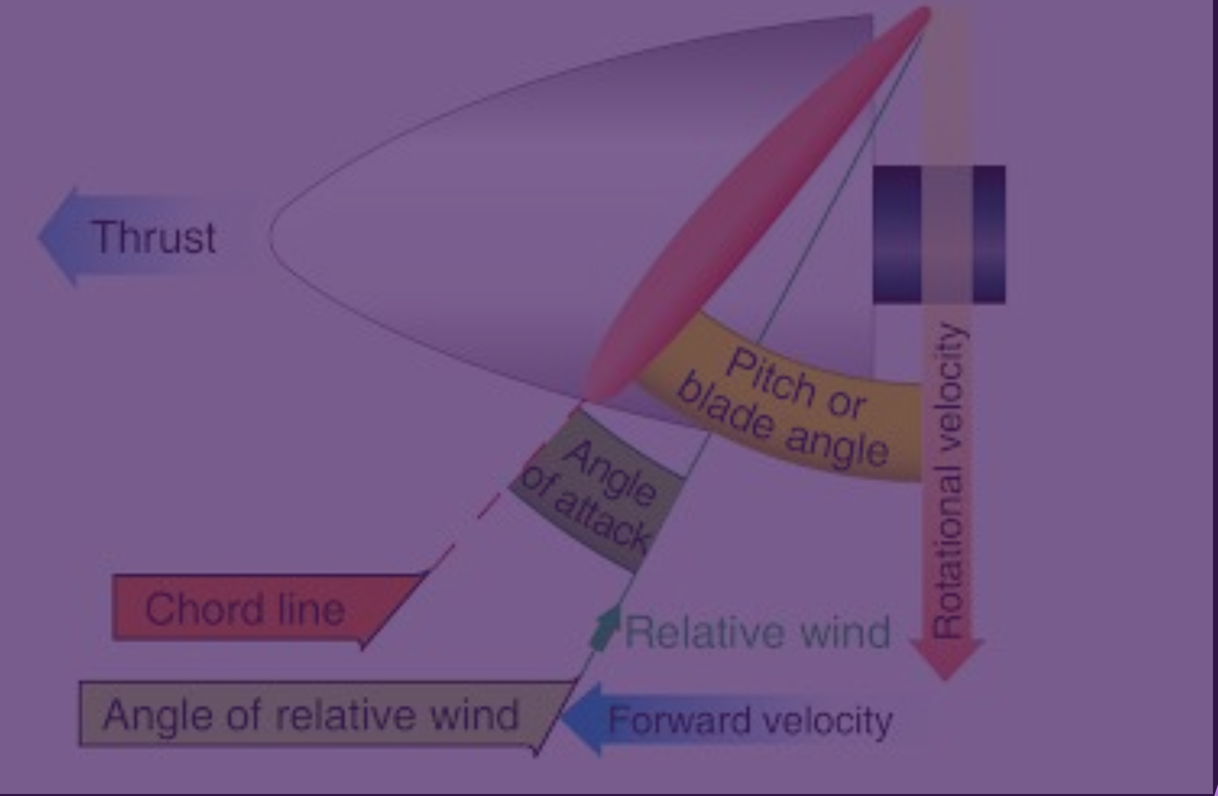

Blades
&
Wind
–
Identical Math
&
Geometry

In the previous chapters a **four-blade propeller** design concept was completed, then explanation of

the motion of skeletal rods

followed that enabled the blades to assume different 3D shapes – that is **morphing**.

In this chapter

1. using the knowledge of how the skeleton with an integrated skin moves
2. an attempt will be made to specify the function that

   governs the blades' 3D geometry

   during the morphing process;
3. having the function it will be compared to the one received earlier, for the radial distribution of the **resulting wind's angles**.

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Előzmény: a Felépítés c. részben kialakítottunk egy **négylapátos légcsavart**, majd a Működés c. részben láttuk

a lapát váz-elemeinek mozgását,

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Abstraction

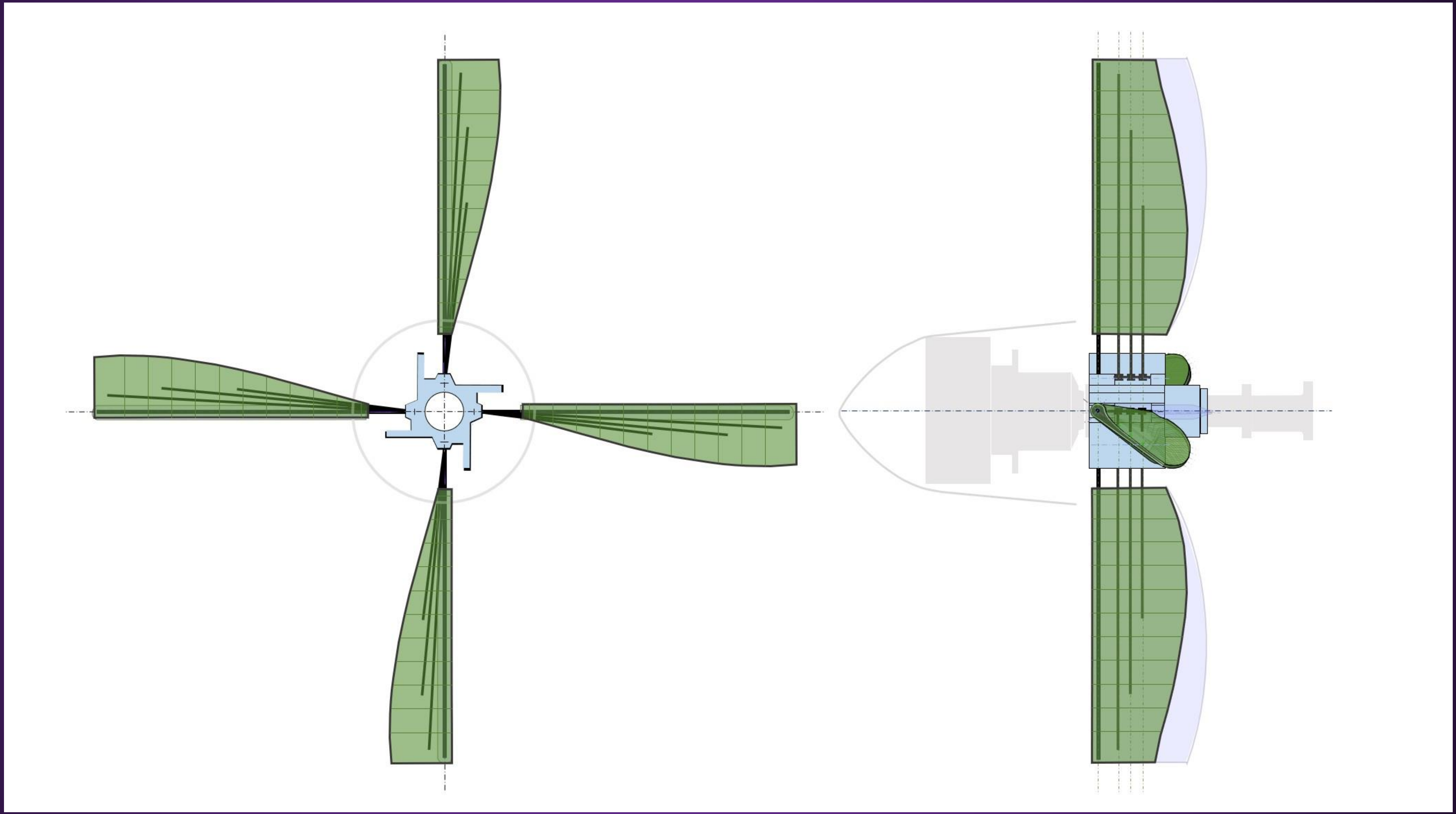
- What we have: a concept design of the blade **mechanism**;
- What we need: a **geometric model** of the blade surface.

Mathematical and geometrical conformity

Concept design of the blade mechanism

- An intermediate stage of blade twist

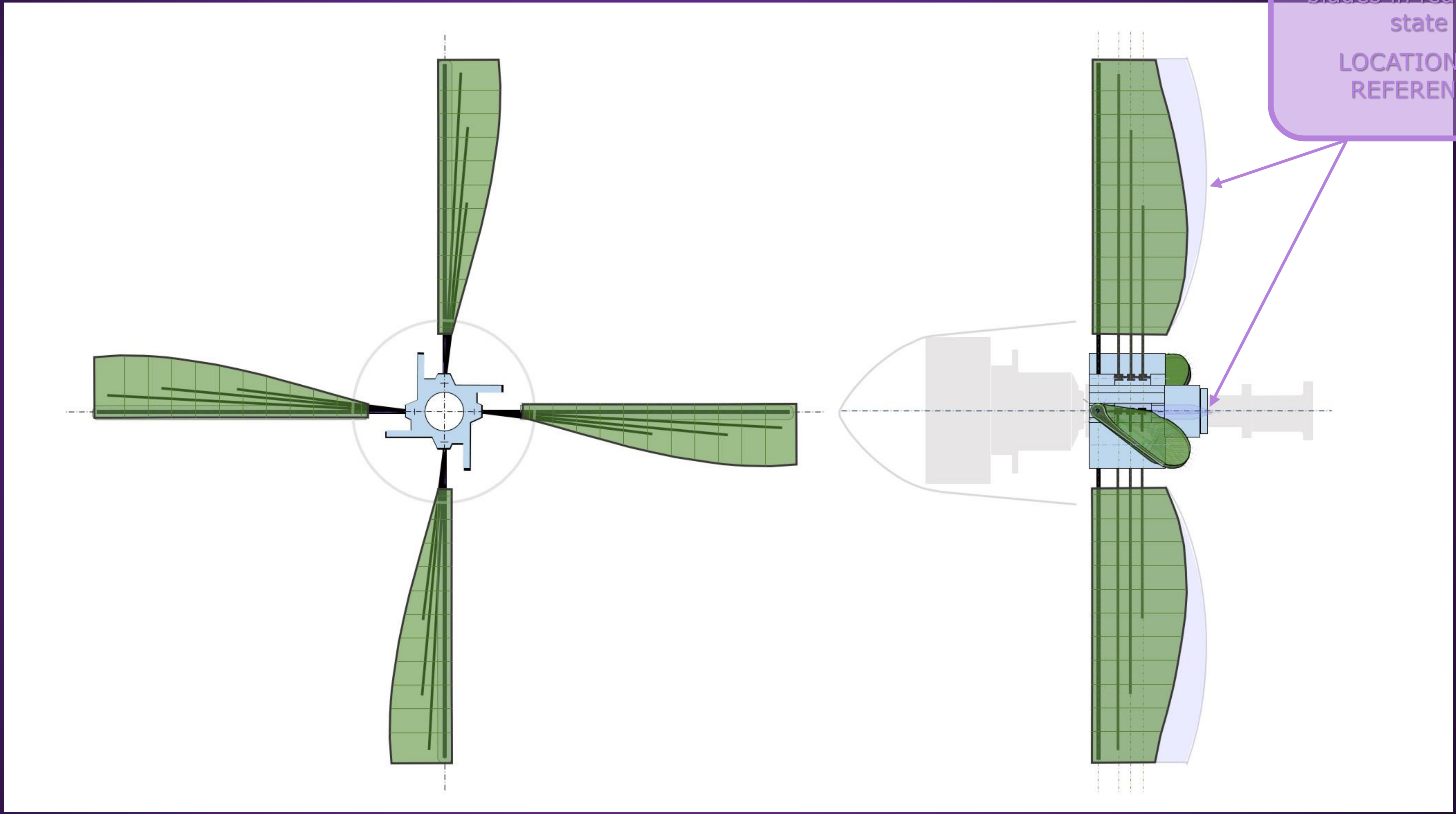
- Geometric model has to be extracted



Mathematical and geometrical conformity

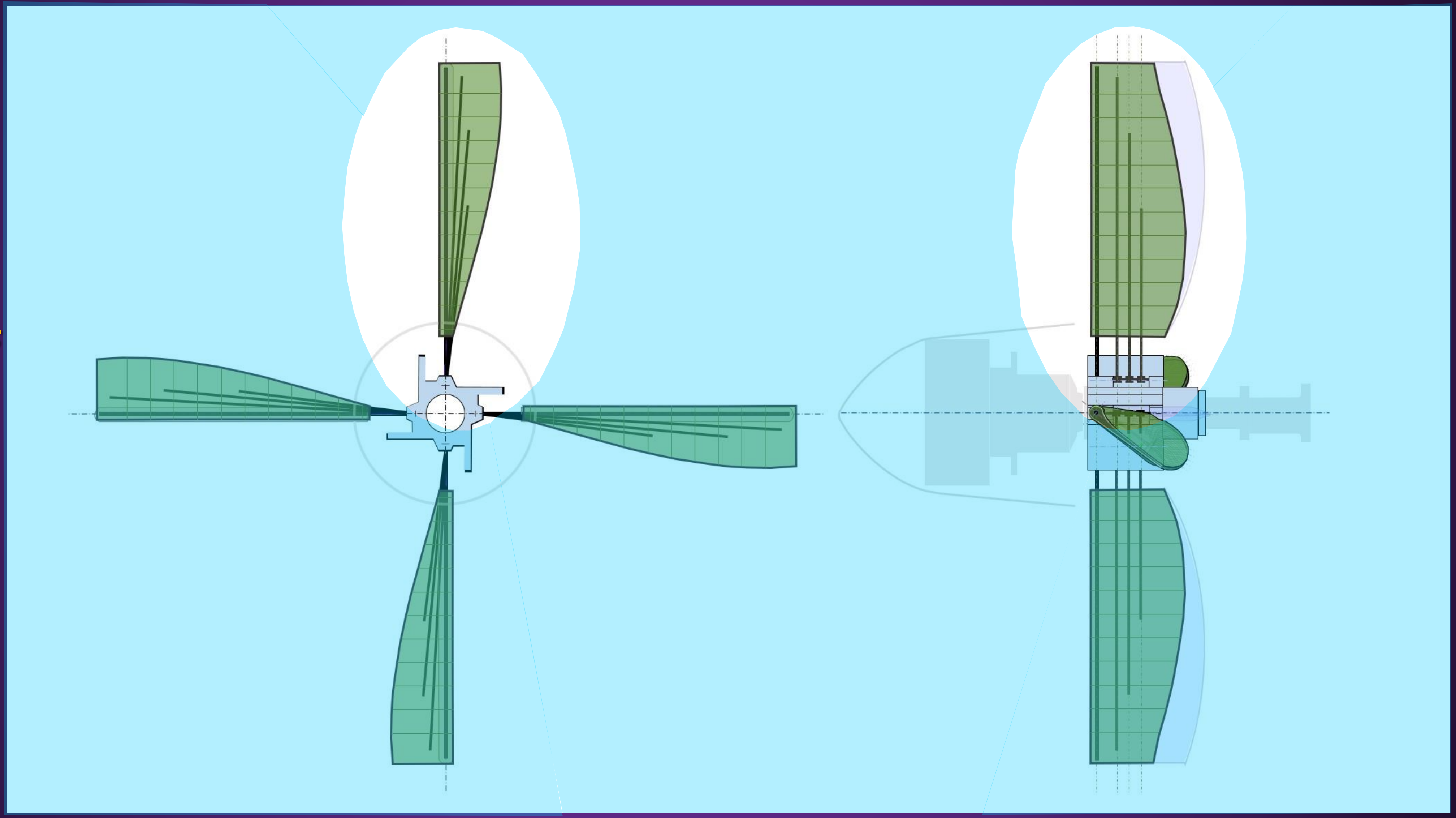
Concept design of the blade mechanism

- An intermediate stage of blade twist
- Geometric model has to be extracted
- Plane of feathered state as location of reference



Mathematical and geometrical conformity

Drawing to model the 3D surface of *this* blade is to be made.



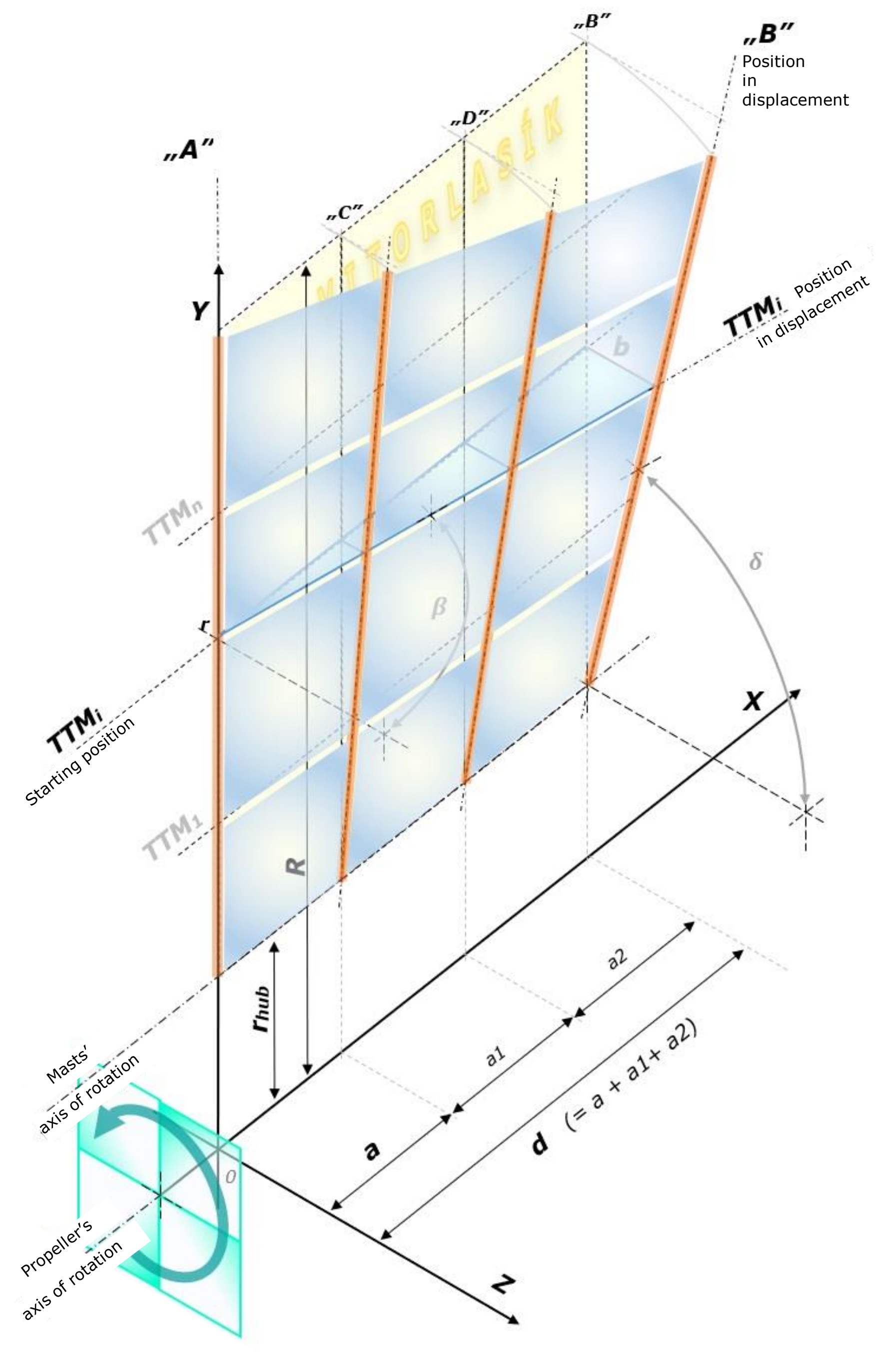
Mathematical and geom

3D model of blade geometry

Identification of reference parts:

- Plane of feathered state (plane X-Y, shaded yellow) ;
- Propeller's plane of rotation (plane Y-Z) and axis of rotation (axis X);
- The masts („A-B-C-D") with their axis of rotation;

  Important detail:
  The propeller's axis of rotation is different from that of the masts**!**
- The **TTM** –s, or more exactly their **medians** (TTM1-i-n);
- The sails (elements of skin).

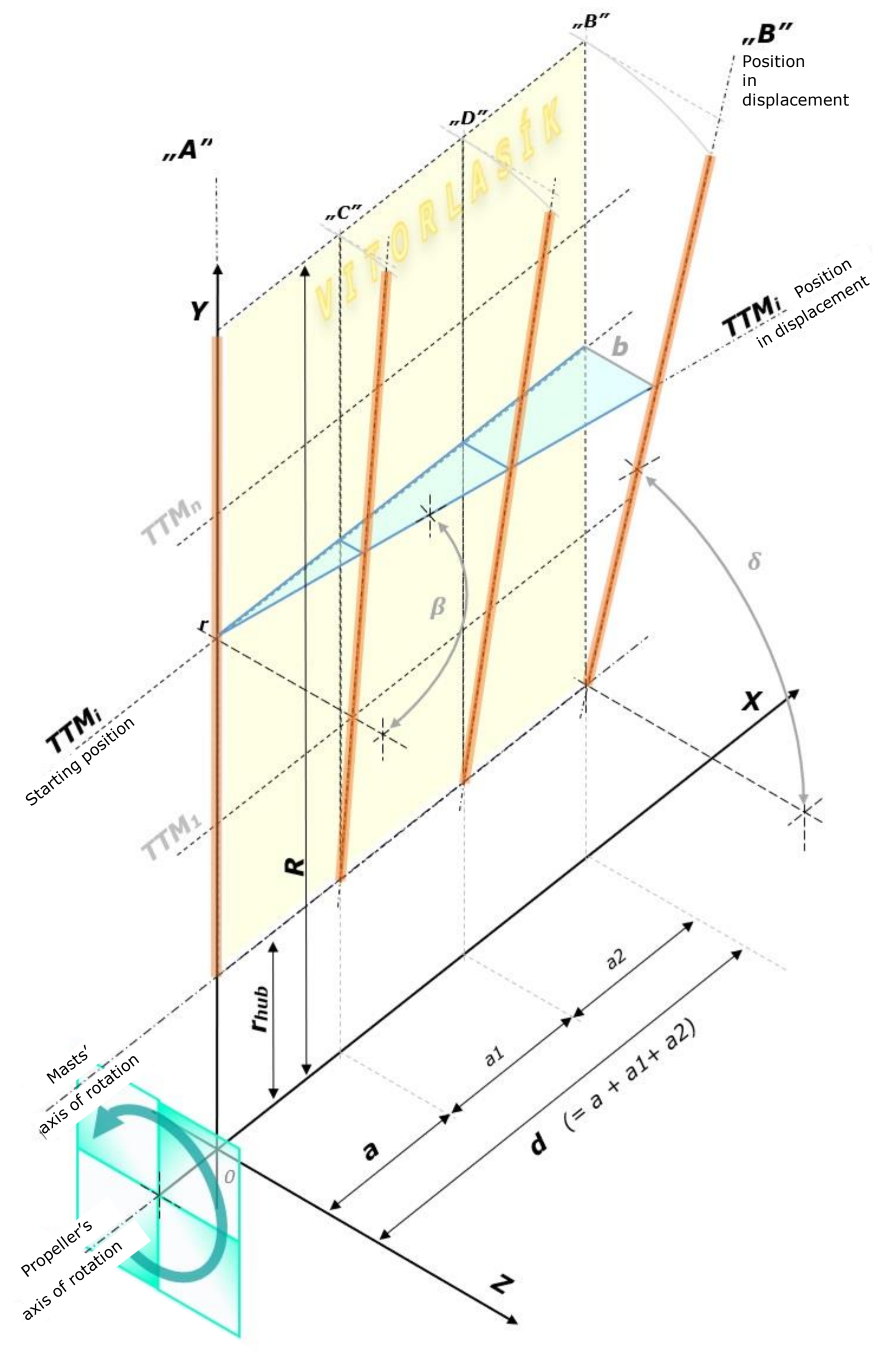


Mathematical and geom

3D model of blade geometry

Sails removed.

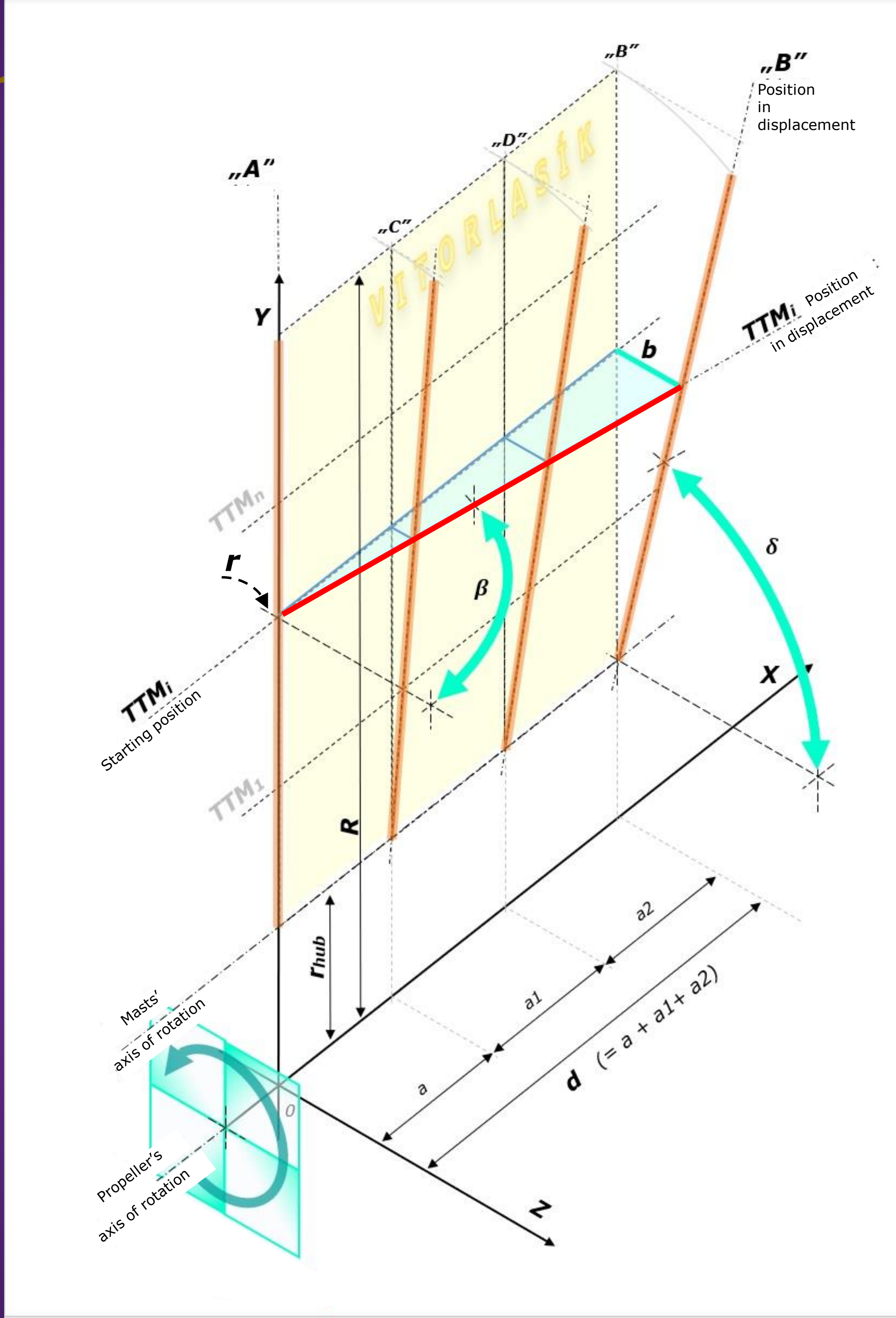
(Skeleton alone is left.)



Mathematical and geom

- A general position (intermediate) TTM is picked
- Be it ***TTM$_i$***
- Centerline of the section belonging to ***TTM$_i$*** is called **typical generator** of the (model-) blade surface
- Two special triangles having one side in common
- Let us write the equation of spatial position of the blade surface using the typical generator ...
- ... in the form of this function:

β = f(r)



Careful :

- Typical generator – straight line constituent of a bent surface – not so trivial!

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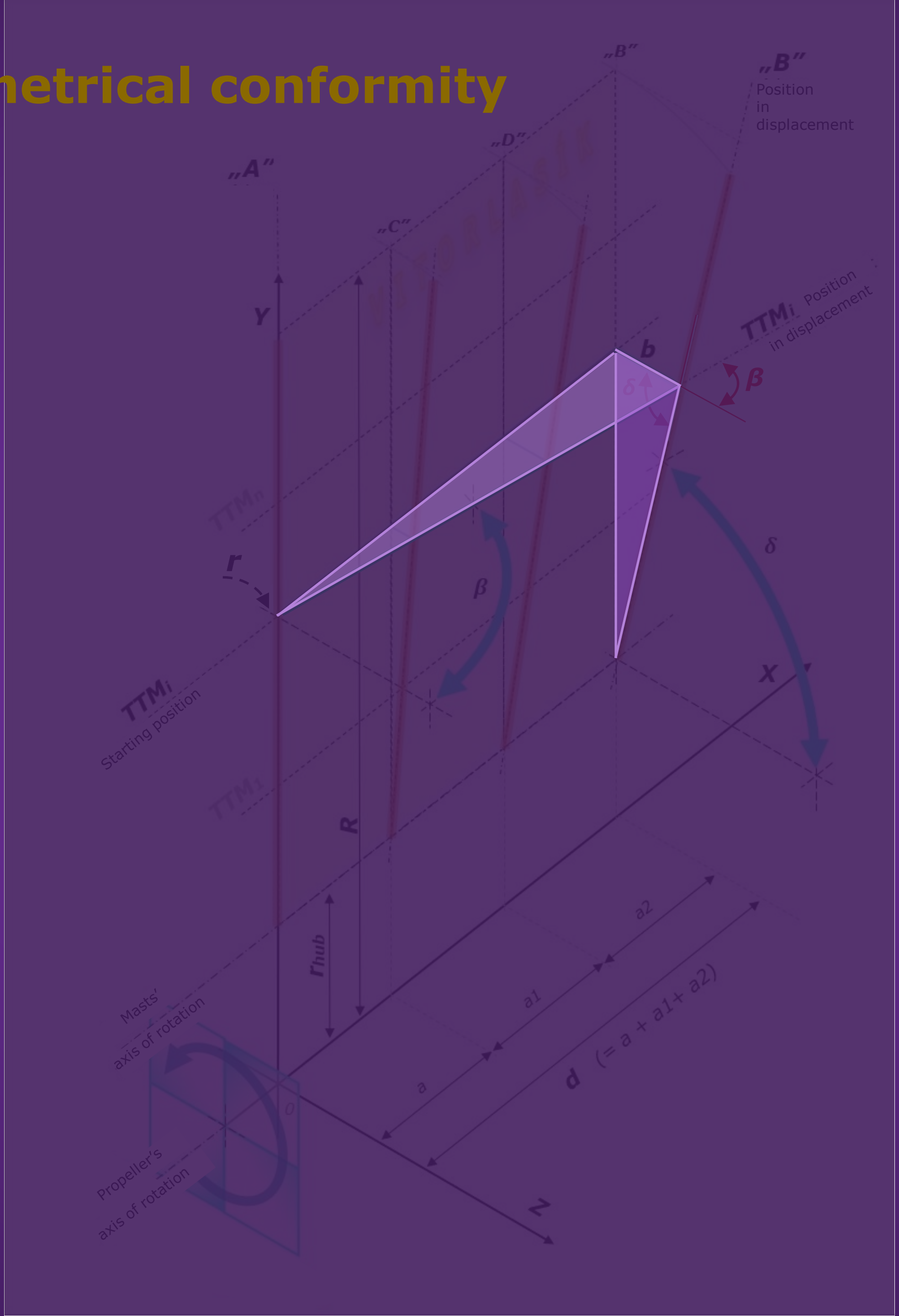

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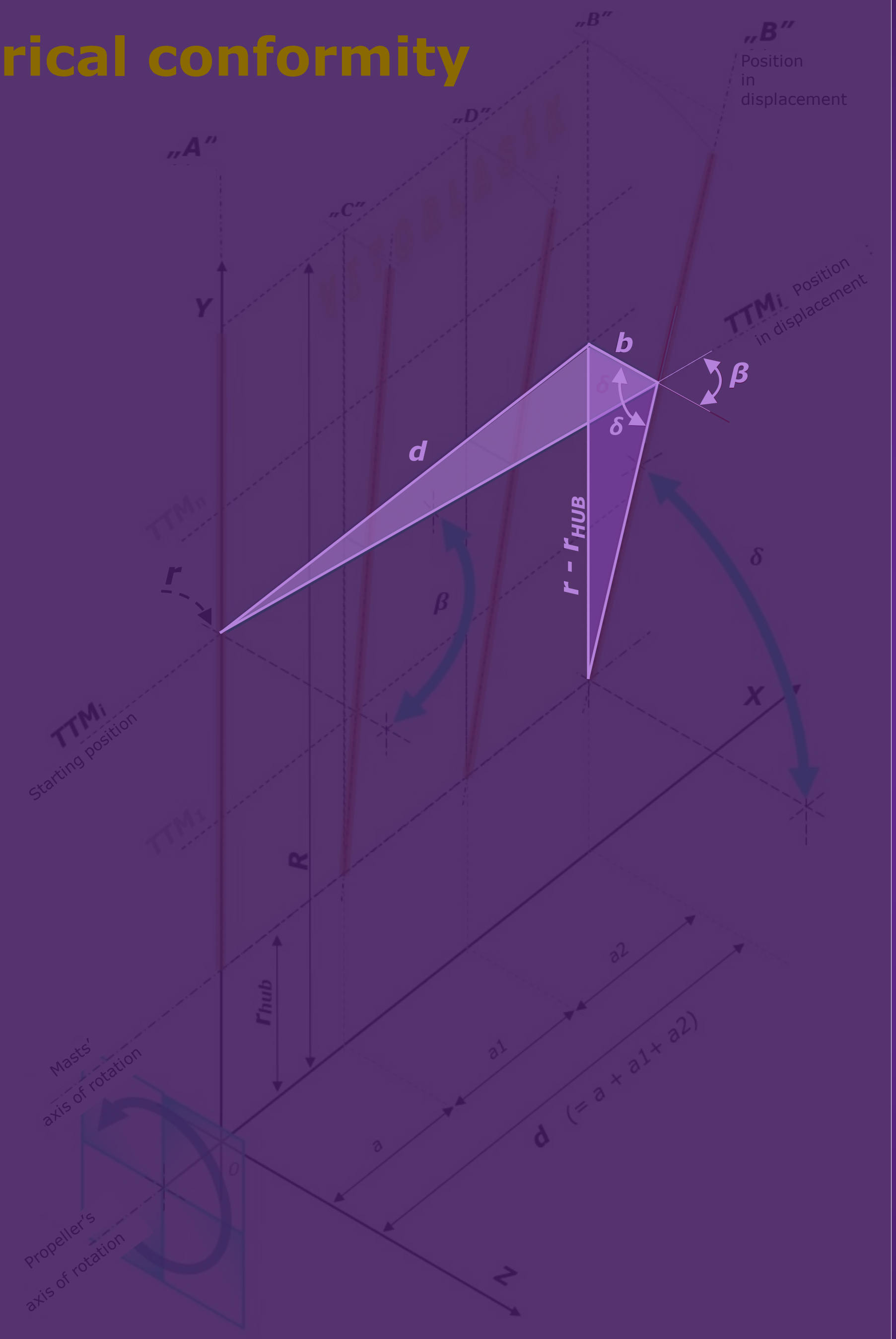

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Mathematical and geometrical conformity

Blade angle β

Expression for blade angle β as function of the radius:

$$b = \frac{d}{tg\,\beta} \quad ; \quad b = \frac{r - rHUB}{tg\,\delta}$$

$$tg\,\beta = \frac{d * tg\,\delta}{r - rHUB}$$

Equation is reshaped to become comparable with that of the resulting wind.
It is known that

$$d,\ tg\,\delta \neq f(r) \quad ,$$

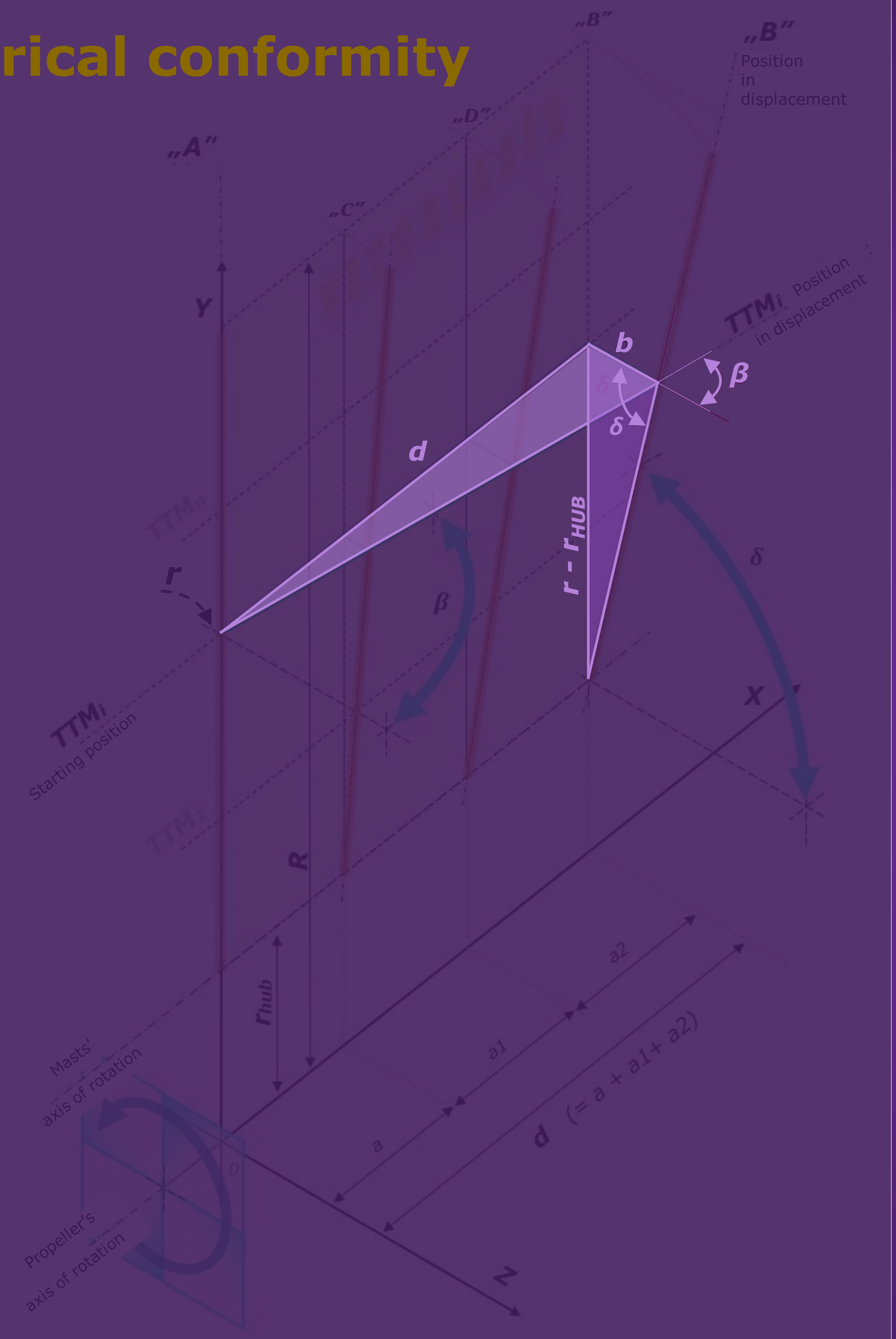
Therefore let's have

$$d * tg\,\delta = C1$$

Now blade angle as function of the radius :

$$tg\,\beta = \frac{C1}{r - rHUB}$$

$$\beta = arctg \frac{C1}{r - rHUB}$$



Angle of resulting wind

Equation we had earlier, for the resulting wind angle :

$$tg\,\gamma = \frac{V_{ax}}{\omega * r}$$

Here too, we transform to prepare for the comparison. It is known that

$$V_{ax}\,,\ \omega \neq f(r) \quad ,$$

Therefore let's have

$$V_{ax} / \omega = C2$$

Then

$$tg\,\gamma = \frac{C2}{r}$$

That is

$$\gamma = arctg \frac{C2}{r}$$

Mathematical and geometrical conformity

Blade angle, β

Angle of resulting wind, γ

$$\beta = arctg\frac{C1}{r - rHUB}$$

$$\gamma = arctg\ \frac{C2}{r}$$

When looking for conformity it is good news the angles of both the blades and the resulting wind are governed by the **same type** of function along the radius :

$$y \sim arctg\ \frac{1}{x}$$

Mathematical and geometrical conformity

- Conformity however is only partial :

$$\beta = arctg\frac{C1}{r - rHUB} \neq \gamma = arctg\ \frac{C2}{r}$$

- Further it will be shown that the difference (or ***error***)

  a) is predictable;

  b) can be expressed in equation and

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Mathematical and geometrical conformity

- Size of the difference between the two functions indicates how close the blade surface comes to the optimal configuration defined by the direction of the resulting wind.

  - For another **example** we will use main data from the previous example, but will consider that the propeller is replaced by a new one of the **torsion blade-** design;
  - It is assumed that at **r = 0,75R** we will have 100% conformity :

$$\beta(0{,}75R) = \gamma(0{,}75R)$$

  - For visual analysis of the error a graph can be built :

$$\Delta = \gamma - \beta = arctg\,\frac{C2}{r} - arctg\,\frac{C1}{r - rHUB}$$

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Mathematical and geometrical conformity

Plan of visual analysis

1) Charts of functions

$$\beta = arctg\frac{d * tg\,\delta}{r - rHUB}$$

and

$$\gamma = arctg\ \frac{V_{ax}}{\omega * r}$$

will be calculated and drawn;

2) Range of calculations:

   r_{HUB} ≤ r ≤ R (R=1000mm)

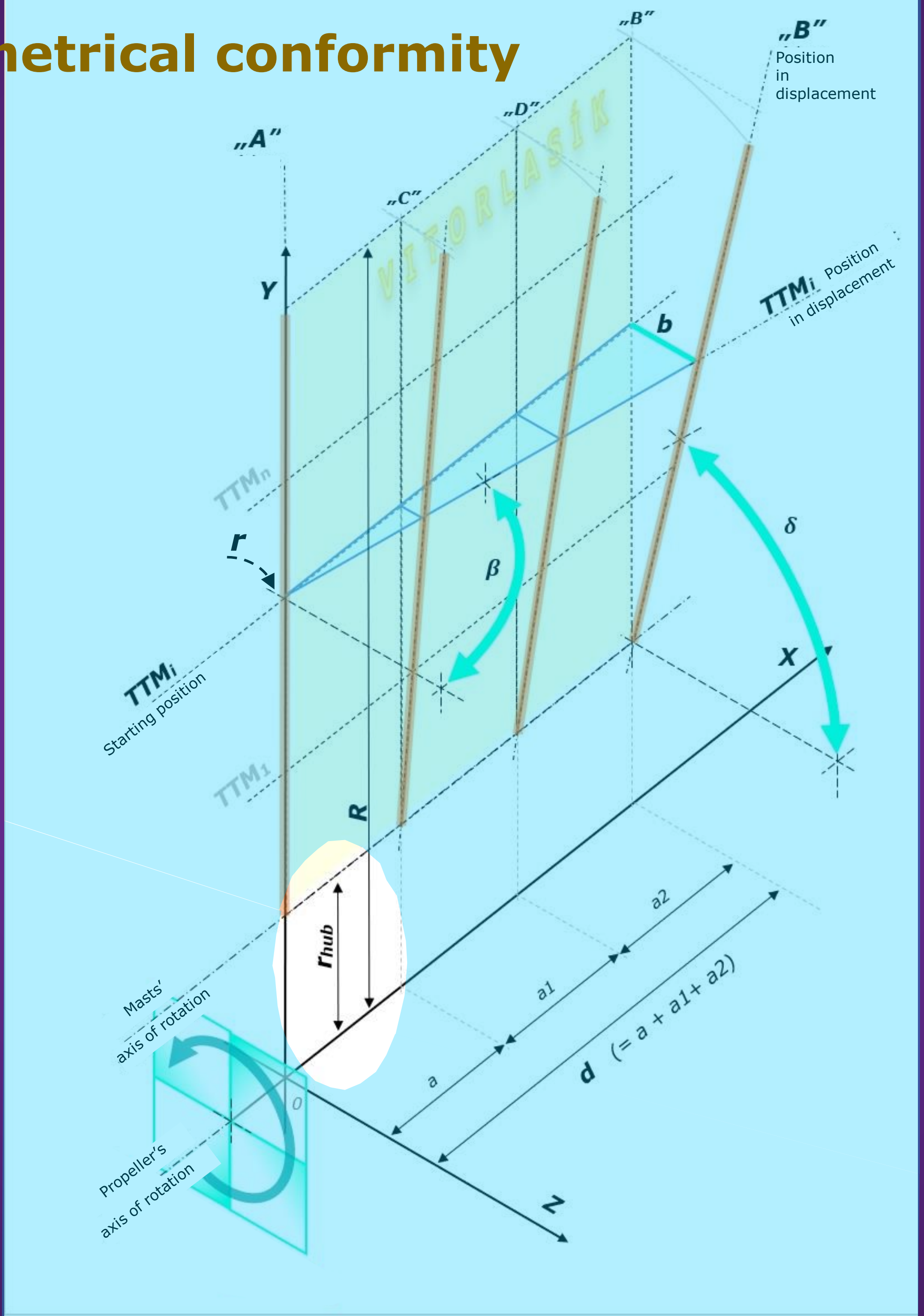
   0 ≤ V_{ax} ≤ 1000km/h

   n = 2598 min^{-1} = 43,3 sec^{-1}

   ω = 2π*n = 272 s^{-1} = ω_{max} =
   = const

   Note
   When calculating charts of the blade angle-function it is made sure they always intersect the resulting wind-charts in the 0,75R points;

3) Degree of conformity of the two charts is manipulated by varying the r_{HUB} value.



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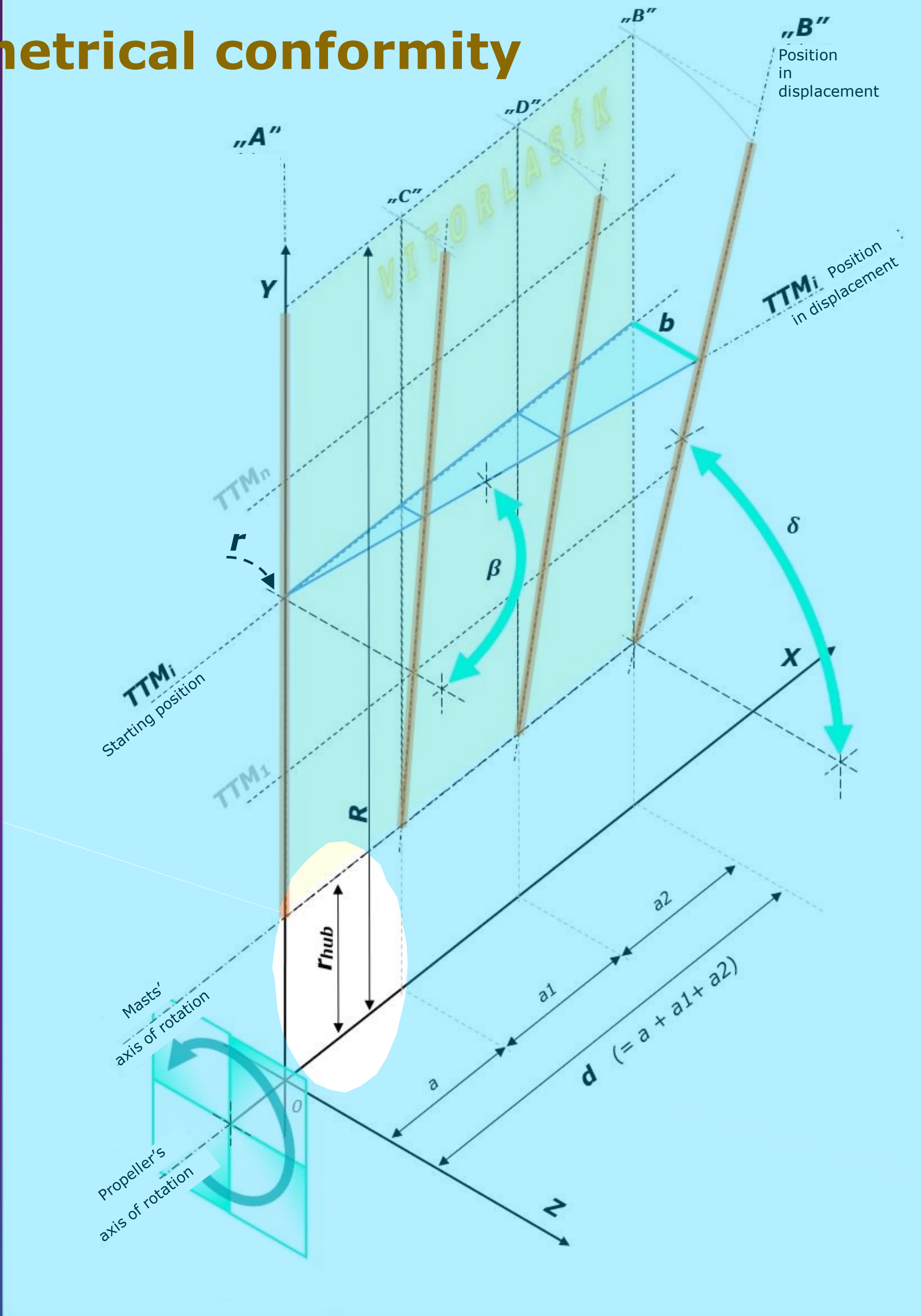
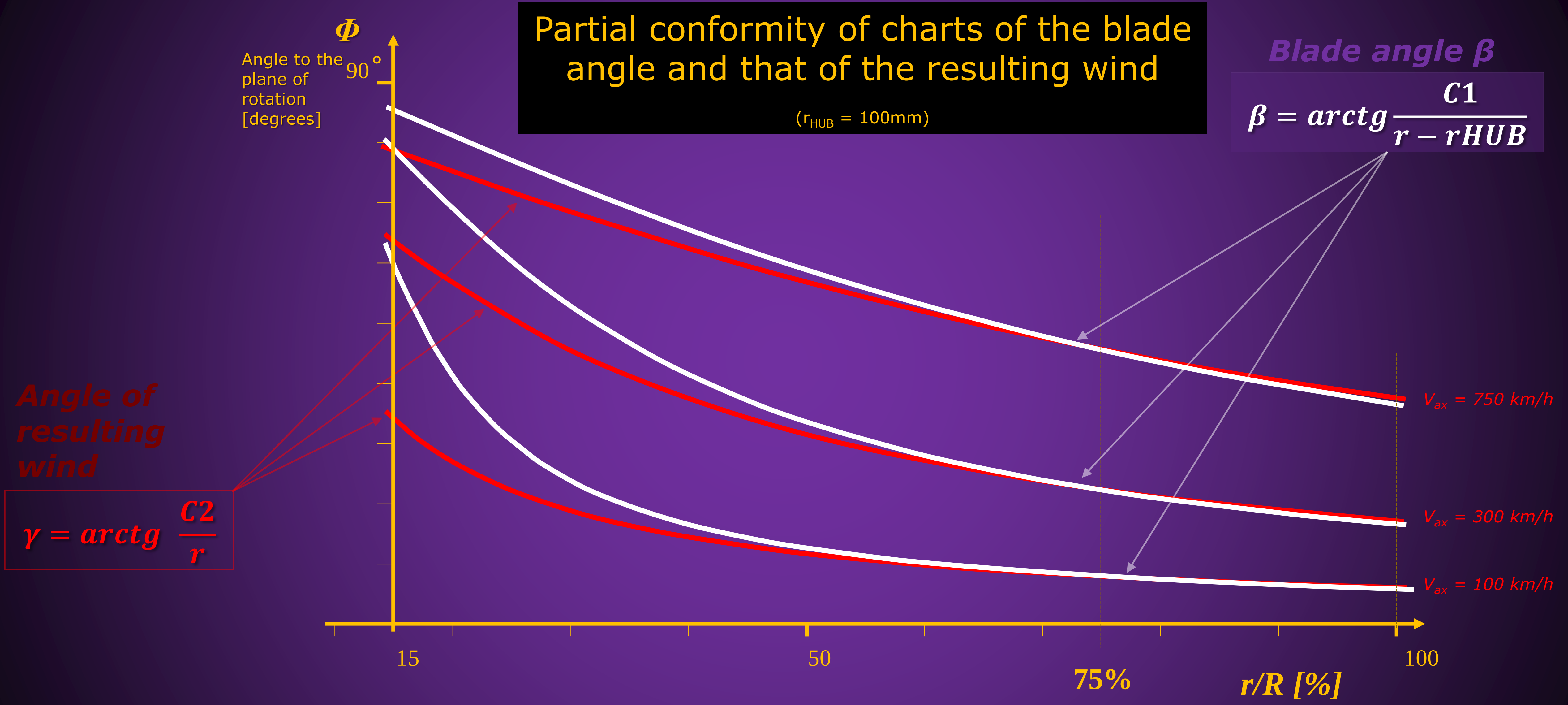


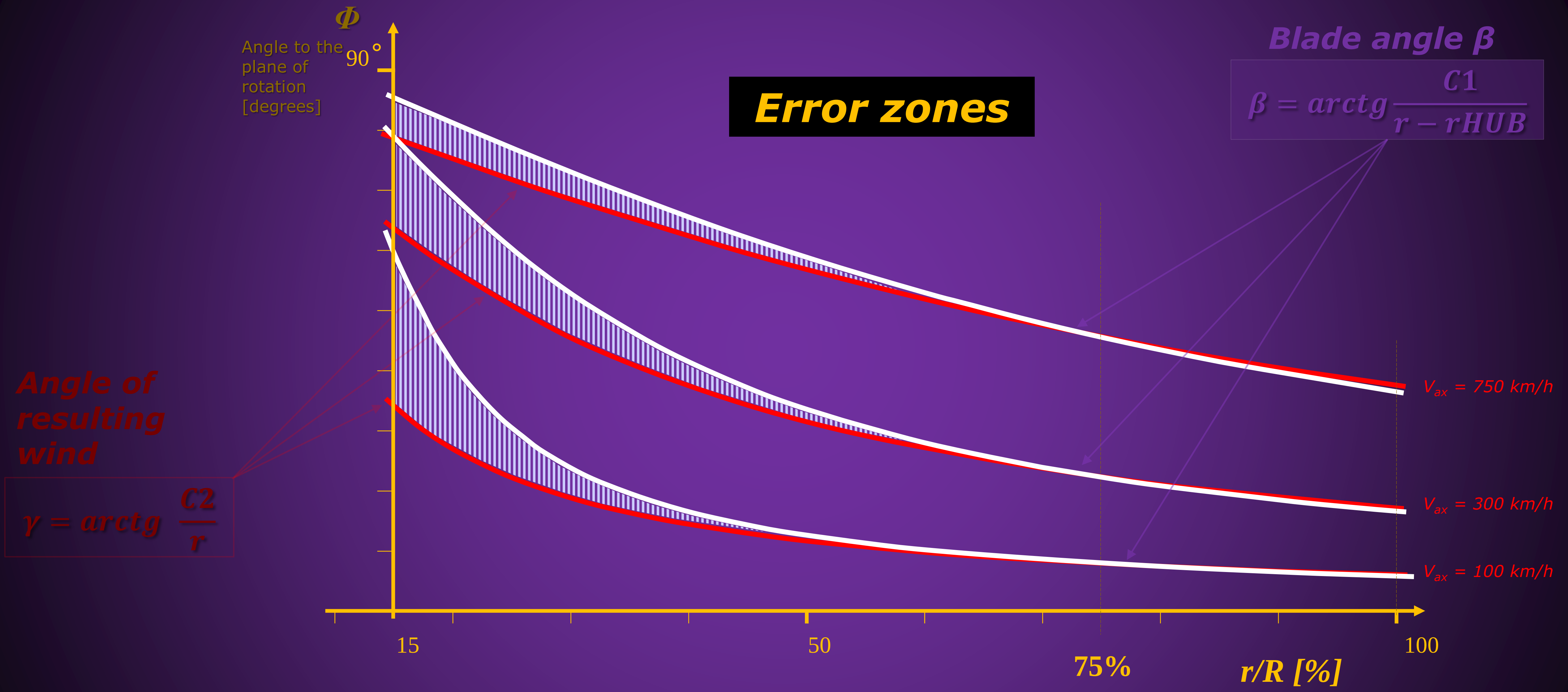
r_{HUB}

- One of the critical parameters of blade construction is r_{HUB} that
- heavily effects twist of the blade surface. By this
- r_{HUB} is main factor in making difference between radial distributions of angles of both the blade and resulting wind.
- The smaller is r_{HUB} , the fuller is conformity between ditributions of angles.
- In case of the CONCENTRIC HUB design r_{HUB} = 0. This makes the error to be zero too.
- So far the ECCENTRIC HUB design has been used to make analysis of the blades' morphing process easier.
- The CONCENTRIC HUB design is introduced in the chapter of DRAWINGS.

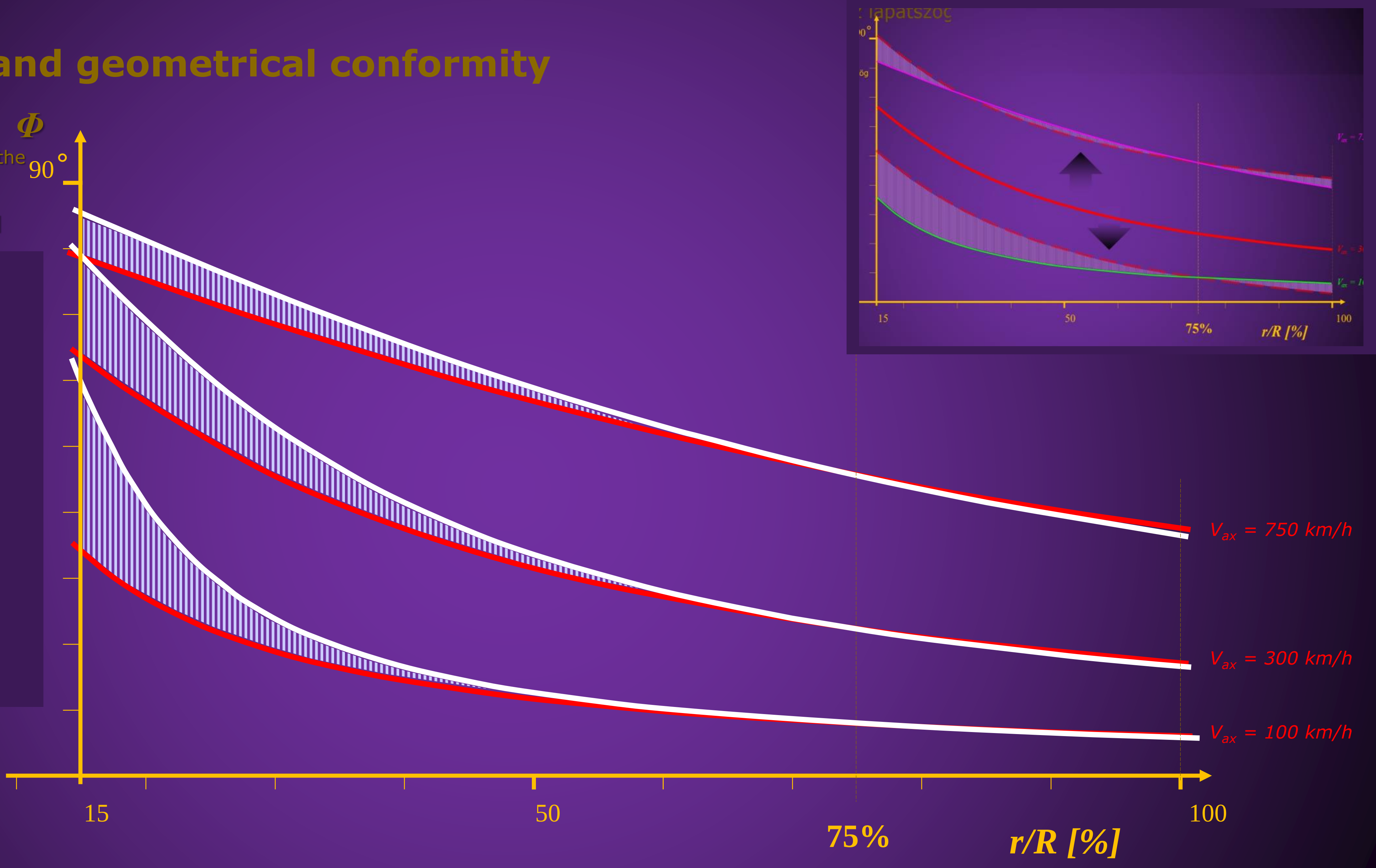
Mathematical and geometrical conformity



Mathematical and geometrical conformity



Mathematical and geometrical conformity



Error zones

- Smaller than before;
- By reducing „r_{HUB}" the errors can be made even smaller;
- At the external third of blade radius (which produces most of the thrust) blade efficiency keepst its maximal value both at high and at low twist;

Mathematical and geometrical conformity

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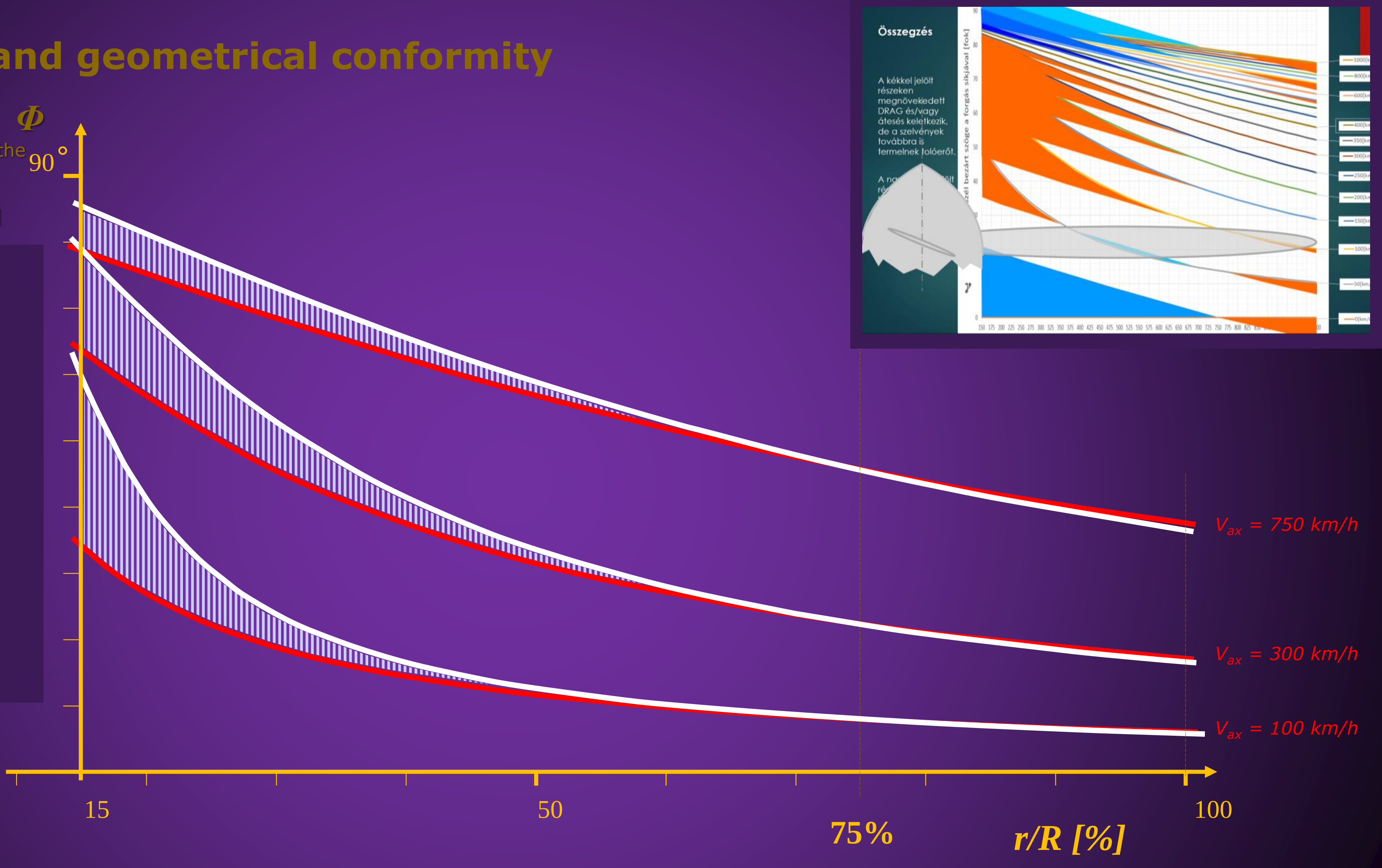
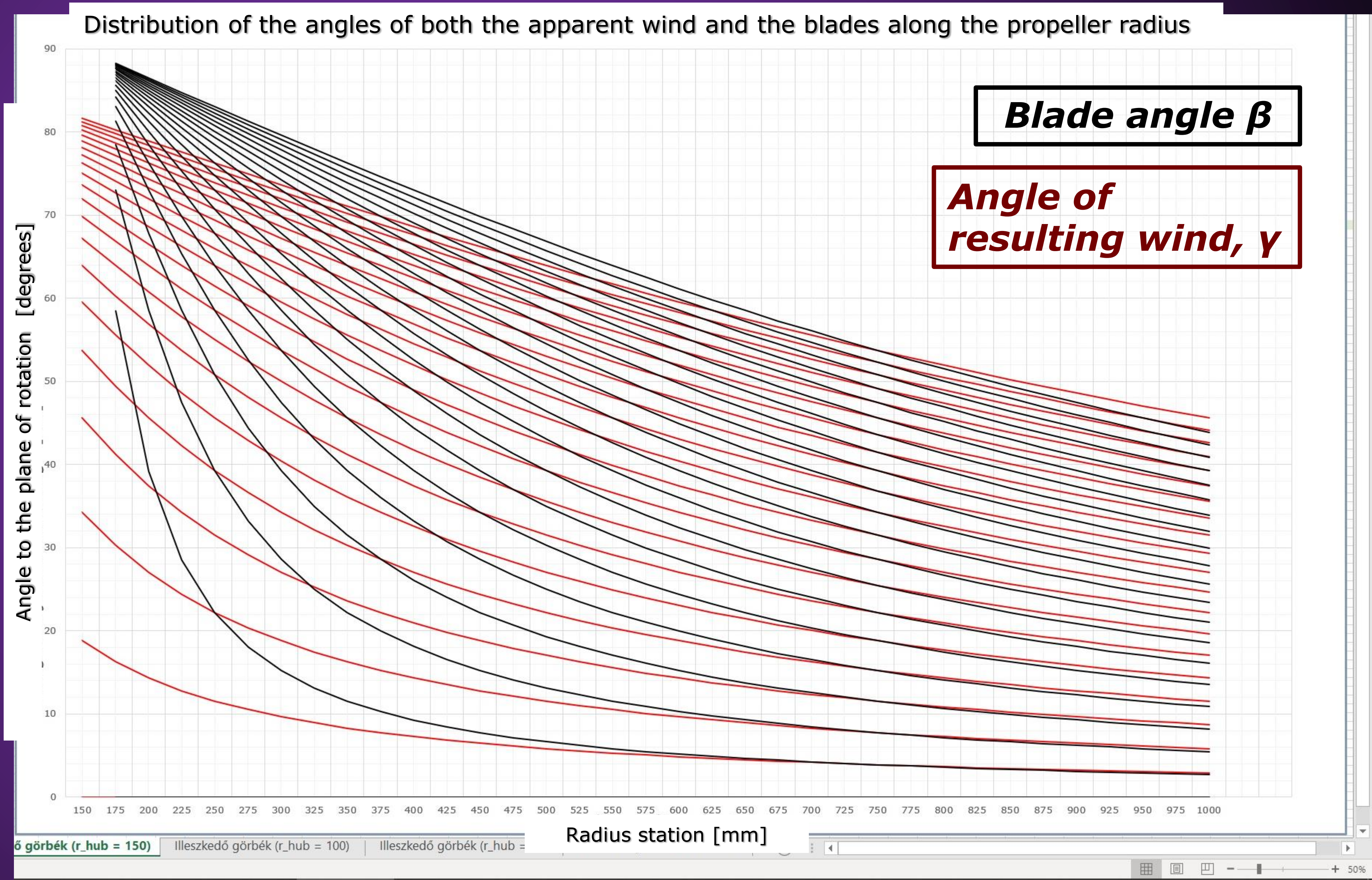


Conformity of functions of both the **blades** and the **resulting wind** can heavily be effected by the size of

$$r_{HUB}$$

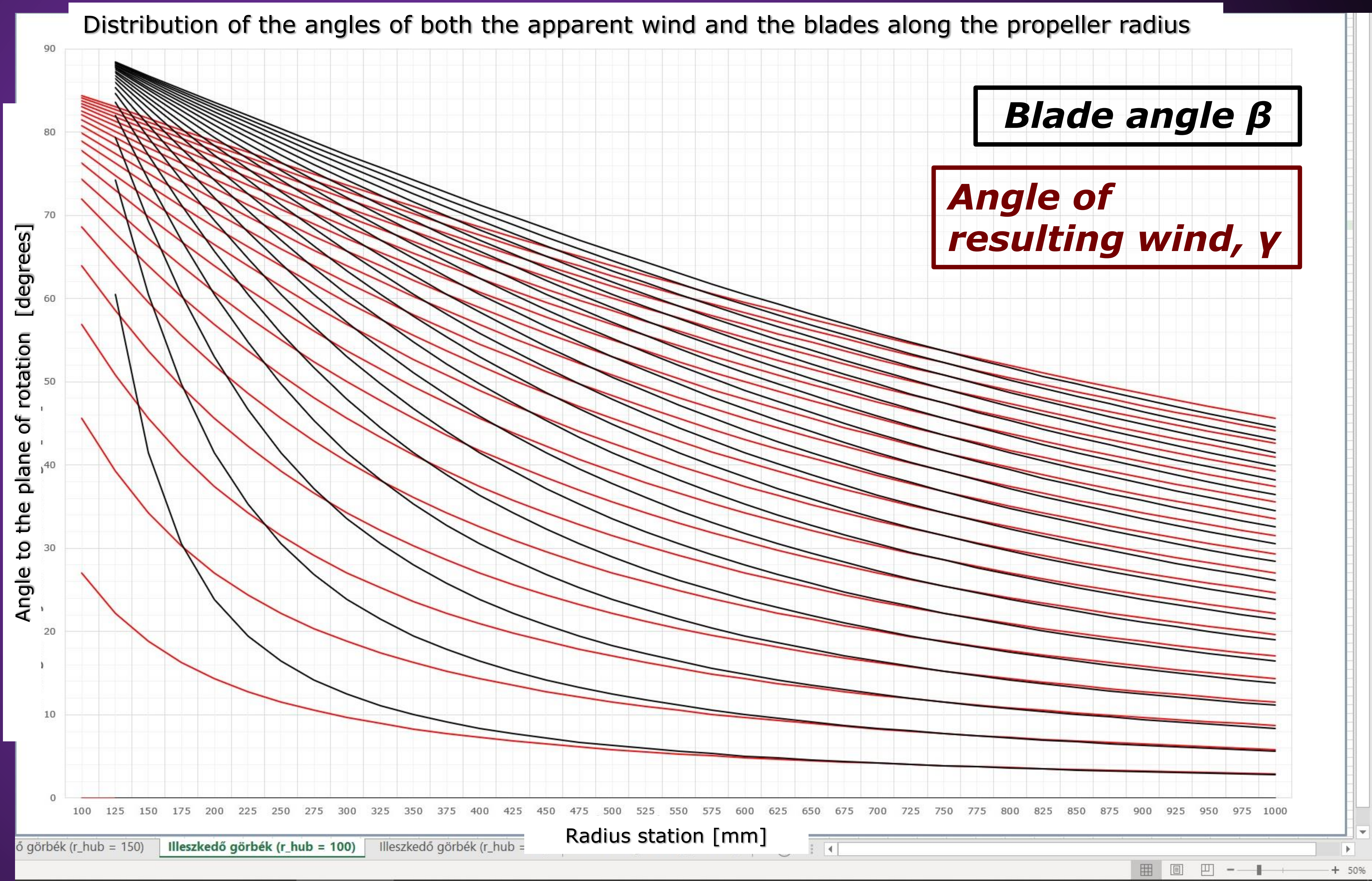
Mathematical and geometrical conformity

R_{HUB} = 150mm



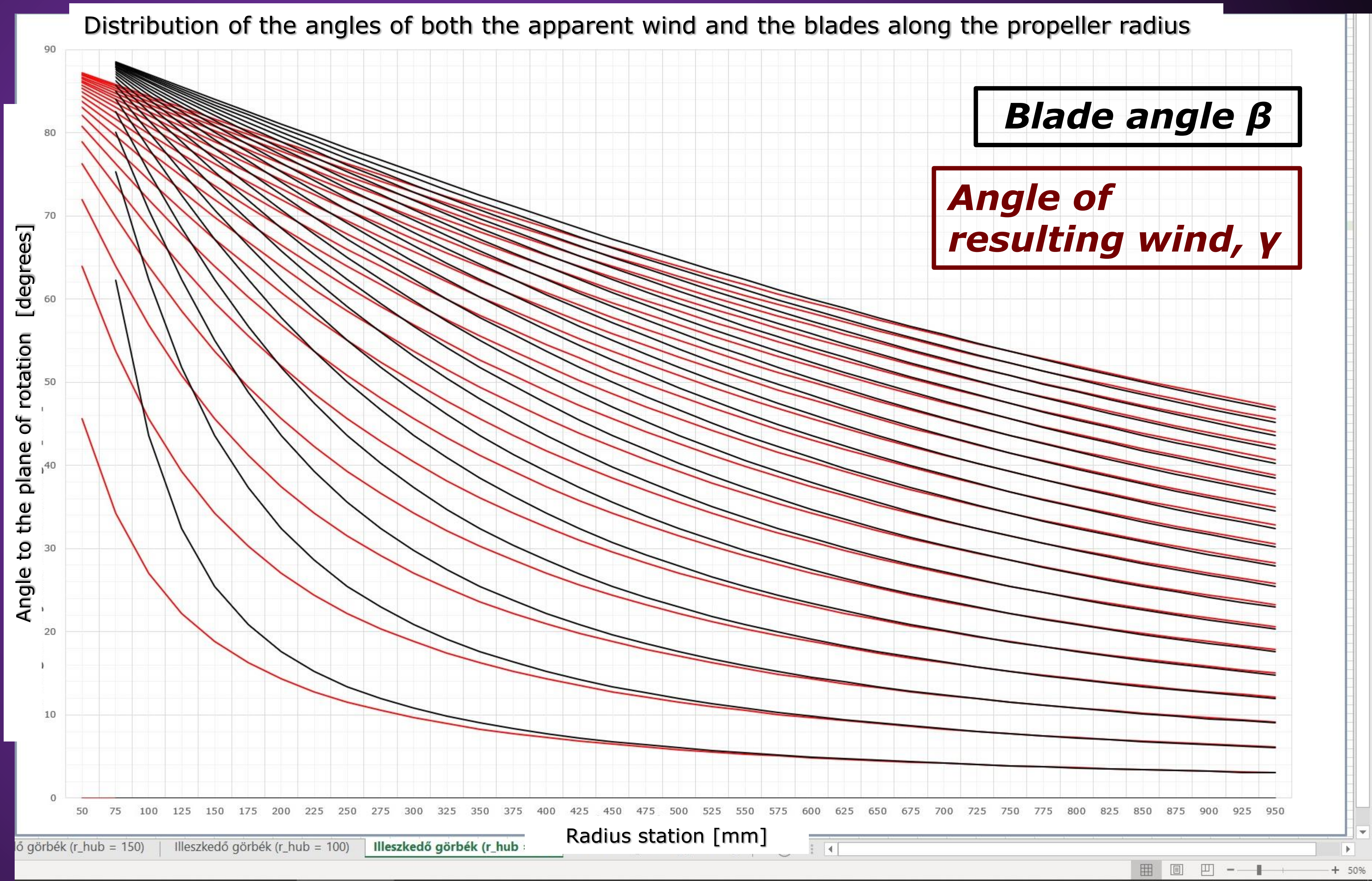
Mathematical and geometrical conformity

R_{HUB} = 100mm



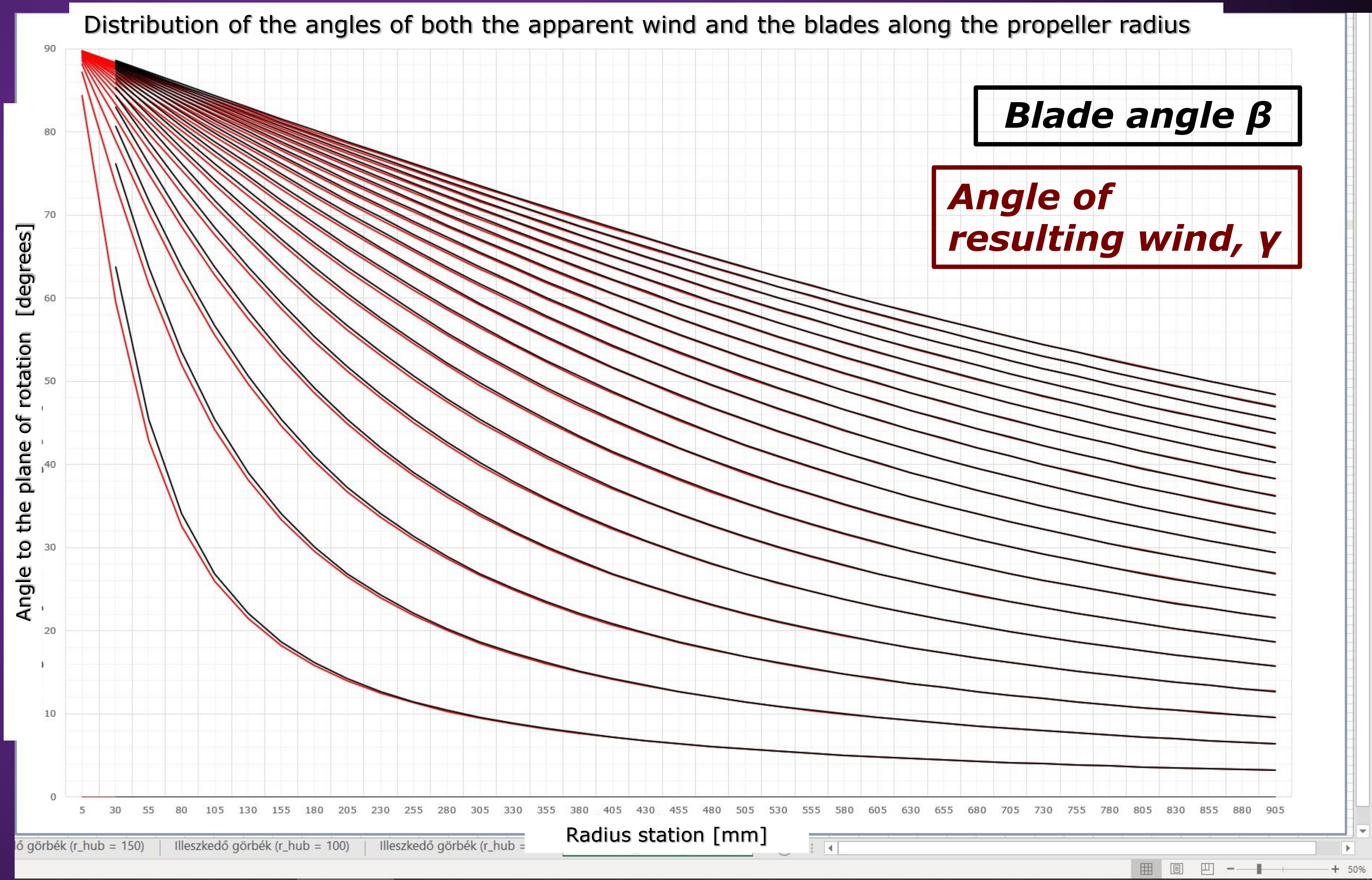
Mathematical and geometrical conformity

R_{HUB} = 50mm



Mathematical and geometrical conformity

R_{HUB} = 5mm



The charts show a steadily improving conformity with the step-by-step reduction of size $\boldsymbol{r_{HUB}}$.

This makes the application of the **CONCENTRIC HUB** design (that allows $\boldsymbol{r_{HUB}} = 0$) most promising.

Mathematical and geometrical conformity